

# 2-4 Study Guide and Intervention

## Deductive Reasoning

**Law of Detachment** Deductive reasoning is the process of using facts, rules, definitions, or properties to reach conclusions. One form of deductive reasoning that draws conclusions from a true conditional  $p \rightarrow q$  and a true statement  $p$  is called the **Law of Detachment**.

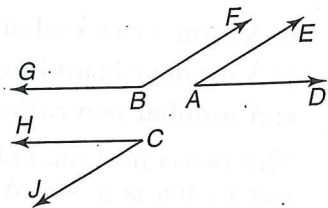
Law of Detachment	If $p \rightarrow q$ is true and $p$ is true, then $q$ is true.
Symbols	$[(p \rightarrow q)] \wedge p \rightarrow q$

**Example** The statement *If two angles are supplementary to the same angle, then they are congruent* is a true conditional. Determine whether each conclusion is valid based on the given information. Explain your reasoning.

- a. Given:  $\angle A$  and  $\angle C$  are supplementary to  $\angle B$ .  
 Conclusion:  $\angle A$  is congruent to  $\angle C$ .

The statement  $\angle A$  and  $\angle C$  are supplementary to  $\angle B$  is the hypothesis of the conditional. Therefore, by the Law of Detachment, the conclusion is true.

Valid



- b. Given:  $\angle A$  is congruent to  $\angle C$ .  
 Conclusion:  $\angle A$  and  $\angle C$  are supplementary to  $\angle B$ .

The statement  $\angle A$  is congruent to  $\angle C$  is not the hypothesis of the conditional, so the Law of Detachment cannot be used. The conclusion is not valid.

Invalid

### Exercises

Determine whether each conclusion is valid based on the true conditional given. If not, write *invalid*. Explain your reasoning.

*If two angles are complementary to the same angle, then the angles are congruent.*

1. Given:  $\angle A$  and  $\angle C$  are complementary to  $\angle B$ .  
 Conclusion:  $\angle A$  is congruent to  $\angle C$ .

Valid

2. Given:  $\angle A \cong \angle C$ .  
 Conclusion:  $\angle A$  and  $\angle C$  are complements of  $\angle B$ .

Invalid; the hypothesis and rule do not match

3. Given:  $\angle E$  and  $\angle F$  are complementary to  $\angle G$ .  
 Conclusion:  $\angle E$  and  $\angle F$  are vertical angles.

Invalid; the conclusions don't match

**2-4**

**Study Guide and Intervention** (continued)

**Deductive Reasoning**

**Law of Syllogism** Another way to make a valid conclusion is to use the **Law of Syllogism**. It is similar to the Transitive Property.

Law of Syllogism	If $p \rightarrow q$ is true and $q \rightarrow r$ is true, then $p \rightarrow r$ is also true.
Symbols	$[(p \rightarrow q)] \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$

**Example** The two conditional statements below are true. Use the Law of Syllogism to find a valid conclusion. State the conclusion.

- (1) If a number is a whole number, ~~then the number is an integer.~~
- (2) ~~If a number is an integer,~~ then it is a rational number.

$p$ : A number is a whole number.  
 $q$ : A number is an integer.  
 $r$ : A number is a rational number.

The two conditional statements are  $p \rightarrow q$  and  $q \rightarrow r$ . Using the Law of Syllogism, a valid conclusion is  $p \rightarrow r$ . A statement of  $p \rightarrow r$  is "if a number is a whole number, then it is a rational number."

**Exercises**

Determine whether you can use the Law of Syllogism to reach a valid conclusion from each set of statements.

- 1. If a dog eats Superdog Dog Food, he will be happy.  
 Rover is happy.

Invalid - No Conclusion

- 2. If an angle is supplementary to an obtuse angle, then it is acute.  
 If an angle is acute, then its measure is less than 90.

If an  $\angle$  is suppl. to an obtuse  $\angle$ , then its measure is less than 90.

- 3. If the measure of  $\angle A$  is less than 90, then  $\angle A$  is acute.  
 If  $\angle A$  is acute, then  $\angle A \cong \angle B$ .

If the  $m\angle A$  is less than 90, then  $\angle A \cong \angle B$ .

- 4. If an angle is a right angle, then the measure of the angle is 90.  
 If two lines are perpendicular, then they form a right angle.

If 2 lines are  $\perp$ , the measure of the  $\angle$  is 90.

- 5. If you study for the test, then you will receive a high grade.  
 Your grade on the test is high.

Invalid - No Conclusion