NAME ______________________ DATE __________ PERIOD _____

2-1 Word Problem Practice

Inductive Reasoning and Conjecture

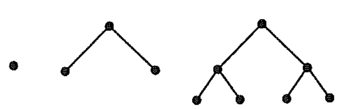
1. RAMPS Rodney is rolling marbles down a ramp. Every second that passes, he measures how far the marbles travel. He records the information in the table shown below.

Second	1st	2nd	3rd	4th
Distance (cm)	20	60	100	140

Make a conjecture about how far the marble will roll in the fifth second.

2. PRIMES A prime number is a number other than 1 that is divisible by only itself and 1. Lucille read that prime numbers are very important in cryptography, so she decided to find a systematic way of producing prime numbers. After some experimenting, she conjectured that $2^n - 1$ is a prime for all whole numbers $n > 1$. Find a counterexample to this conjecture.

3. GENEOLOGY Miranda is developing a chart that shows her ancestry. She makes the three sketches shown below. The first dot represents herself. The second sketch represents herself and her parents. The third sketch represents herself, her parents, and her grandparents.

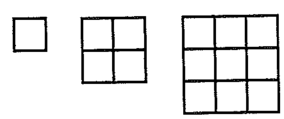


Sketch what you think would be the next figure in the sequence.

4. MEDALS Barbara is in charge of the award medals for a sporting event. She has 31 medals to give out to various individuals on 6 competing teams. She asserts that at least one team will end up with more than 5 medals. Do you believe her assertion? If you do, try to explain why you think her assertion is true, and if you do not, explain how she can be wrong.

PATTERNS For Exercises 5–7, use the following information.

The figure shows a sequence of squares each made out of identical square tiles.



5. Starting from zero tiles, how many tiles do you need to make the first square? How many tiles do you have to add to the first square to get the second square? How many tiles do you have to add to the second square to get the third square?

6. Make a conjecture about the list of numbers you started writing in your answer to Exercise 5.

7. Make a conjecture about the sum of the first n odd numbers.

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2-1 Enrichment

Counterexamples

When you make a conclusion after examining several specific cases, you have used **inductive reasoning**. However, you must be cautious when using this form of reasoning. By finding only one **counterexample**, you disprove the conclusion.

Example **Is the statement $\frac{1}{x} \leq 1$ true when you replace x with 1, 2, and 3? Is the statement true for all reals? If possible, find a counterexample.**

$\frac{1}{1} = 1$, $\frac{1}{2} < 1$, and $\frac{1}{3} < 1$. But when $x = \frac{1}{2}$, then $\frac{1}{x} = 2$. This counterexample shows that the statement is not always true.

Exercises

1. The coldest day of the year in Chicago occurred in January for five straight years. Is it safe to conclude that the coldest day in Chicago is always in January?

2. Suppose John misses the school bus four Tuesdays in a row. Can you safely conclude that John misses the school bus every Tuesday?

3. Is the equation $\sqrt{k^2} = k$ true when you replace k with 1, 2, and 3? Is the equation true for all integers? If possible, find a counterexample.

4. Is the statement $2x = x + x$ true when you replace x with $\frac{1}{2}$, 4, and 0.7? Is the statement true for all real numbers? If possible, find a counterexample.

5. Suppose you draw four points A, B, C, and D and then draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$. Does this procedure give a quadrilateral always or only sometimes? Explain your answers with figures.

6. Suppose you draw a circle, mark three points on it, and connect them. Will the angles of the triangle be acute? Explain your answers with figures.

Lesson 2-1